Math 1B -- §5.1 – 6.2 Test – Spring '10   Name______________________________

Show your work for credit. Write all responses on separate paper. Do not use a calculator.

1. Suppose your bicycle has a speedometer and you want to use it to estimate the distance you travel on the bike by observing the velocity at 5 second intervals, recorded in the following table:

Time (s)	0	5	10	15	20	25	30
Velocity (Km/hr)	9	18	27	31	36	45	50

In order to have the time and the velocity in consistent units, you convert the velocity reading to meters/sec (1 Km/Hr = 5/18 m/s)

Time (s)	0	5	10	15	20	25	30
Velocity (m/s)	2.5	5	7.5	8.6			

   a. Finish the table by approximating the velocities $v(20)$, $v(25)$ and $v(30)$ to the nearest tenth of a meter per second.
   b. Use right endpoints as sample points to evaluate the distance the bicycle travelled during the first 15 seconds. Indicate the units of your approximation.
   c. Use left endpoints as sample points to evaluate the distance the bicycle travelled during the first 15 seconds. Indicate the units of your approximation.

2. The area A of the region R that lies under the graph of $f(x)=\dfrac{\ln(x)}{x}$ for $3 \le x \le 7$ is the limit of the sum of the areas of approximating rectangles

   $A=\lim_{n\to\infty} R_n = \lim_{n\to\infty}\left(f(x_1)\Delta x + f(x_2)\Delta x + \cdots + f(x_n)\Delta x\right)$

   where the partition $3 = x_0 < x_1 < \cdots < x_n = 7$ contains subintervals of equal length.

   a. Express x_i in terms of i and n, express Δx in terms of n and simplify the sigma form for the Riemann sum R_n as much as possible without evaluating it.
   b. Use the fundamental theorem of calculus (II) to evaluate the limit $A=\lim_{n\to\infty} R_n$.

3. Use midpoints as sample points to compute $\int_1^2 x\,dx$ as the limit of Riemann sums.

   Recall that $\sum_{i=1}^{n} i = \dfrac{n(n+1)}{2}$,

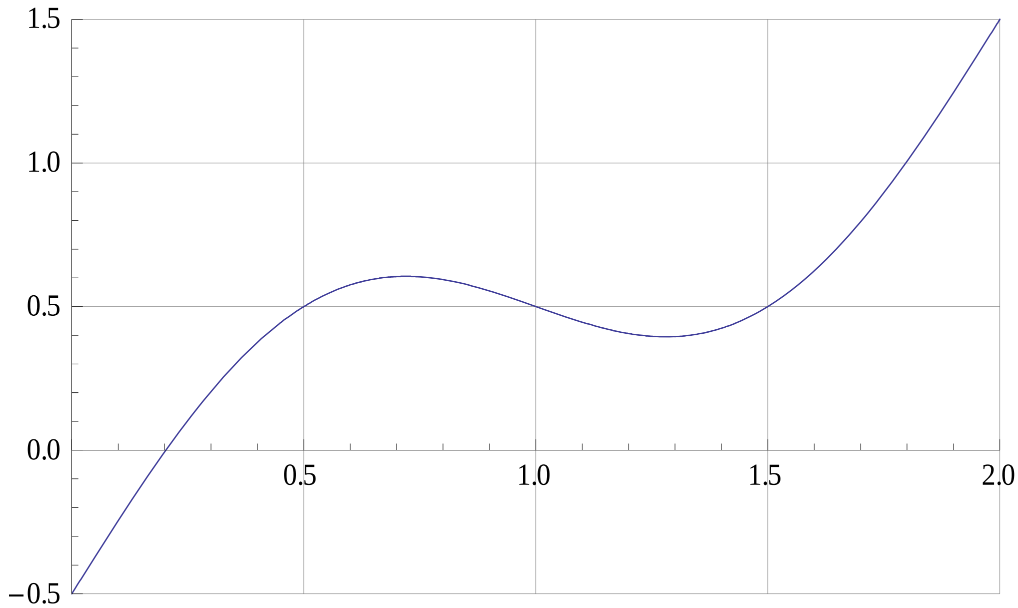
4. Sketch a graph for the region whose area is equal to $\lim_{n\to\infty}\dfrac{\pi}{8}\sum_{i=1}^{n}\tan\left(\dfrac{\pi}{8}+\dfrac{i\pi}{8n}\right)$ and write that area as an integral in the form $\int_a^b f(x)\,dx$. You don't need to evaluate the limit or compute the area.

5. Evaluate the integral $\int_{\pi/6}^{\pi/4}\sin(2x+\pi)\,dx$ using the fundamental theorem of calculus.

6. Simplify $\dfrac{d}{du}\int_u^1 \sin\left(y^2\right)dy$ using the fundamental theorem of calculus.

7. Simplify $\dfrac{d}{dx}\int_0^{\sin x} e^{-t^2}\,dt$ using the fundamental theorem of calculus.

8. Consider the graph of $f(t)$ shown below and let $F(x)=\int_0^x f(t)\,dt$:



a. Use the graph to give approximate values for each the following

i. $F(0.2)$

ii. $F(0.5)$

iii. $F(0.7)$

iv. $F(1)$

v. $F(1.3)$

vi. $F(1.5)$

vii. $F(2)$

b. What local minimum value(s) does $F(x)$ achieve on the interval [0,2] ?

c. What local maximum value(s) does $F(x)$ achieve on the interval [0,2] ?

d. What, if any, inflection points does $F(x)$ have?

e. What is $F'(1)$?

9. Set up integrals to compute the area of the triangle with vertices (1,1), (1,4) and (3,2). You don't need to evaluate the integrals, but it may help to verify they are correct.

10. Find the volume of the solid obtained by rotating about the x-axis the region under the curve $f(x)=\sqrt{x}$ from $x = 0$ to $x = 1$.

Math 1B -- §5.1 – 6.2 Test Solutions – Spring '10

1. Suppose your bicycle has a speedometer and you want to use it to estimate the distance you travel on the bike by observing the velocity at 5 second intervals, recorded in the following table:

Time (s)	0	5	10	15	20	25	30
Velocity (Km/hr)	9	18	27	31	36	45	50

In order to have the time and the velocity in consistent units, you convert the velocity reading to meters/sec (1 Km/Hr = 5/18 m/s)

Time (s)	0	5	10	15	20	25	30
Velocity (m/s)	2.5	5	7.5	8.6	10	12.5	13.9

a. Finish the table by approximating the velocities $v(20)$, $v(25)$ and $v(30)$ to the nearest tenth of a meter per second.
SOLN: Multiply times by 5/18 to produce the m/s values for velocity.

b. Use right endpoints as sample points to evaluate the distance the bicycle travelled during the first 15 seconds. Indicate the units of your approximation.
SOLN: Since each time interval has length 5 sec, we can factor that out and compute 5(5+7.5+8.6) = 5*21.1 =105.5 meters.

c. Use left endpoints as sample points to evaluate the distance the bicycle travelled during the first 15 seconds. Indicate the units of your approximation.
SOLN: 5(0+5+7.5) = 5*12.5 = 62.5 meters.

2. The area A of the region R that lies under the graph of $f(x)=\frac{\ln(x)}{x}$ for $3\le x\le 7$ is the limit of the sum of the areas of approximating rectangles
$A=\lim_{n\to\infty}R_n=\lim_{n\to\infty}\left(f(x_1)\Delta x+f(x_2)\Delta x+\cdots+f(x_n)\Delta x\right)$
where the partition $3=x_0<x_1<\cdots<x_n=7$ contains subintervals of equal length.

a. Express x_i in terms of i and n, express Δx in terms of n and simplify the sigma form for the Riemann sum R_n as much as possible without evaluating it.

SOLN: $\Delta x=\frac{7-3}{n}=\frac{4}{n}$ so that $x_i=a+i\Delta x=3+\frac{4i}{n}$ and $R_n=\frac{4}{n}\sum_{i=1}^{n}\frac{\ln\left(3+\frac{4i}{n}\right)}{3+\frac{4i}{n}}=4\sum_{i=1}^{n}\frac{\ln\left(\frac{3n+4i}{n}\right)}{3n+4i}$

b. Use the fundamental theorem of calculus (II) to evaluate the limit $A=\lim_{n\to\infty}R_n$.

$$\int_3^7\frac{\ln x}{x}dx=\int_{\ln 3}^{\ln 7}u\,du=\frac{1}{2}\left((\ln 7)^2-(\ln 3)^2\right)\text{ where }u=\ln x\Rightarrow du=\frac{dx}{x}$$

3. Use midpoints as sample points to compute $\int_1^2 x\,dx$ as the limit of Riemann sums.

Recall that $\sum_{i=1}^{n}i=\frac{n(n+1)}{2}$.

SOLN:

$$\int_1^2 x\,dx=\lim_{n\to\infty}\frac{1}{n}\sum_{i=1}^{n}\left(1+\frac{i}{n}-\frac{1}{2n}\right)=\lim_{n\to\infty}\frac{1}{n}\left(\sum_{i=1}^{n}1+\frac{1}{n}\sum_{i=1}^{n}i+\frac{1}{2n}\sum_{i=1}^{n}1\right)=\lim_{n\to\infty}\frac{1}{n}\left(n+\frac{n^2+n}{2n}+\frac{1}{2}\right)=\lim_{n\to\infty}1+\frac{1}{2}+\frac{1}{n}=\frac{3}{2}$$

4. Sketch a graph for the region whose area is equal to $\lim_{n\to\infty}\frac{\pi}{8n}\sum_{i=1}^{n}\tan\left(\frac{\pi}{8}+\frac{i\pi}{8n}\right)$ and write that area as an integral in the form $\int_a^b f(x)\,dx$. You don't need to evaluate the limit or compute the area.

SOLN: Evidently, $\Delta x=\frac{b-a}{n}=\frac{\pi}{8n}$ so $b-a=\frac{\pi}{8}$. Also, it seems that

$x_i=a+i\Delta x=\frac{\pi}{8}+\frac{i\pi}{8n}\Rightarrow a=\frac{\pi}{8}\Rightarrow b=\frac{\pi}{4}$ and so $\lim_{n\to\infty}\frac{\pi}{8n}\sum_{i=1}^{n}\tan\left(\frac{\pi}{8}+\frac{i\pi}{8n}\right)=\int_{\pi/8}^{\pi/4}\tan(x)\,dx$

5. Evaluate the integral $\int_{\pi/6}^{\pi/4}\sin(2x+\pi)\,dx$ using the fundamental theorem of calculus.

SOLN: $\int_{\pi/6}^{\pi/4}\sin(2x+\pi)\,dx=\int_{\pi/6}^{\pi/4}-\sin(2x)\,dx=\frac{1}{2}\cos(2x)\Big|_{\pi/6}^{\pi/4}=\frac{1}{2}\left(0-\frac{1}{2}\right)=-\frac{1}{4}$

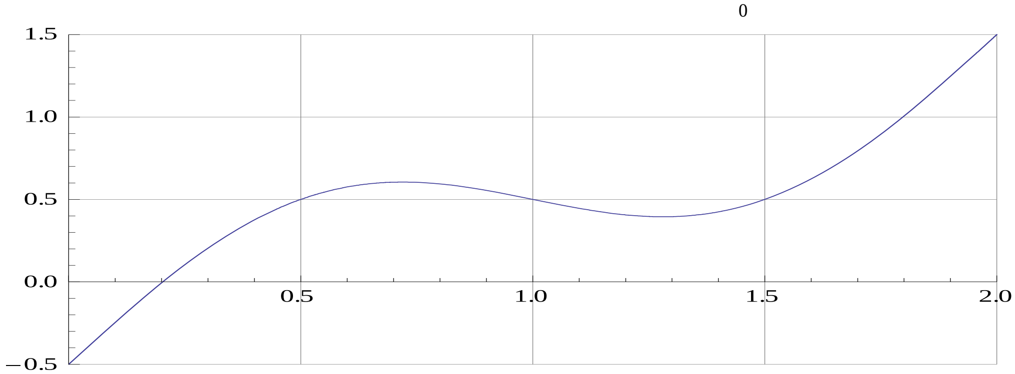
6. Simplify $\frac{d}{du}\int_u^1\sin\left(y^2\right)dy$ using the fundamental theorem of calculus.

SOLN: $\frac{d}{du}\int_u^1\sin\left(y^2\right)dy=-\frac{d}{du}\int_1^u\sin\left(y^2\right)dy=\sin\left(u^2\right)$

7. Simplify $\frac{d}{dx}\int_0^{\sin x}e^{-t^2}\,dt$ using the fundamental theorem of calculus.

SOLN: $\frac{d}{dx}\int_0^{\sin x}e^{-t^2}\,dt=\frac{du}{dx}\frac{d}{du}\int_0^u e^{-t^2}\,dt=e^{-\sin^2 x}\cos x$

8. Consider the graph of $f(t)$ shown below and let $F(x)=\int_0^x f(t)\,dt$:



a. Use the graph to give approximate values for each the following

i. $F(0.2)\approx -0.05$

ii. $F(0.5)\approx -0.05+0.08=0.03$

iii. $F(0.7)\approx 0.03+0.12=0.15$

iv. $F(1)\approx 0.15+0.18=0.33$

v. $F(1.3)\approx 0.33+0.13=0.46$

vi. $F(1.5)\approx 0.46+0.09=0.55$

vii. $F(2)\approx 0.55+0.45=1.00$

b. What local minimum value(s) does $F(x)$ achieve on the interval [0,2] ?
SOLN: Clearly $F(0.2)\approx -0.05$ is a local minimum.

c. What local maximum value(s) does $F(x)$ achieve on the interval [0,2] ?
SOLN: $F(0)=0$ is a local max and $F(2)\approx 1.00$ is a global max on [0,2]

d. What, if any, inflection points does $F(x)$ have?

SOLN: On $(0, 0.2)$, F is decreasing at a decreasing rate (concave up). On $(0.2, 0.7)$, F is increasing at an increasing rate (concave up). On $(0.7, 1.3)$ F is increasing at a decreasing rate (concave down). On $(1.3, 2.0)$ F is increasing at an increasing rate. Thus F changes concavity at $(0.7, F(0.7))$ and again at $(1.3, F(1.3))$, which are the two inflection points.

e. What is $F'(1)$?

SOLN: $F'(1) = f(1) = 0.5$

9. Set up integrals to compute the area of the triangle with vertices (1,1), (1,4) and (3,2). You don't need to evaluate the integrals, but it may help to verify they are correct.

SOLN: $\int_1^3 5 - x - \left(\frac{1}{2}x + \frac{1}{2}\right) dx = \int_1^3 \frac{9}{2} - \frac{3x}{2}\, dx = \frac{9}{2}x - \frac{3}{4}x^2 \Big|_1^3 = \frac{27}{2} - \frac{27}{4} - \frac{9}{2} + \frac{3}{4} = 3$

10. Find the volume of the solid obtained by rotating about the x-axis the region under the curve $f(x) = \sqrt{x}$ from $x = 0$ to $x = 1$.

$\pi \int_0^1 \left(\sqrt{x}\right)^2 dx = \frac{\pi x^2}{2} \Big|_0^1 = \frac{\pi}{2}$